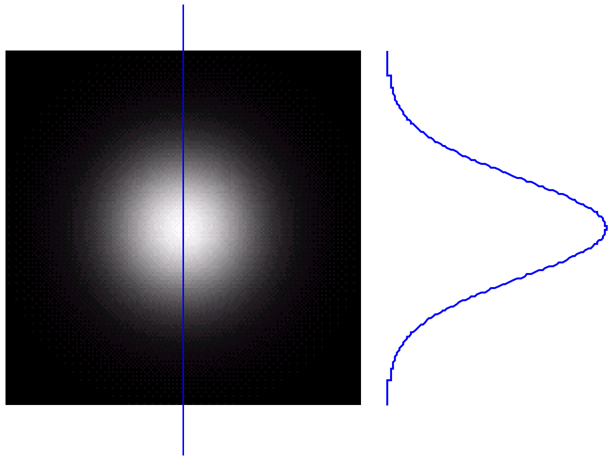
Gaussian Beams

The output beam from most lasers is Gaussian in shape. This means that the intensity profile follows a distribution of the form:

$$G(r) = A \cdot \exp^{\frac{-2 \cdot r^2}{w_o{}^2}} \tag{1}$$

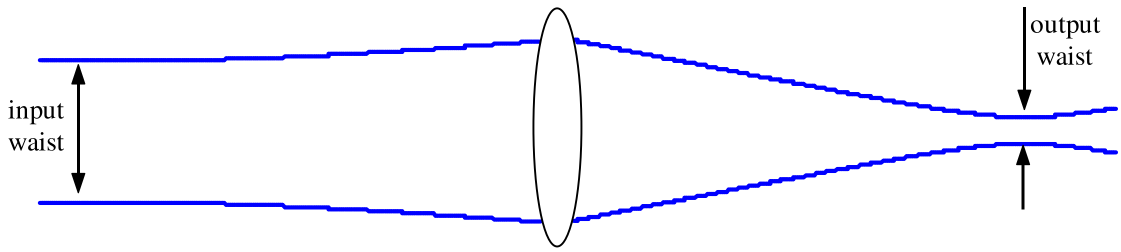
where: A represents the peak height at the centre
r is a radial distance,
w_o is the beam radius at the $1/e^2$ intensity level, where $2w_o$ is known as the beam 'waist'.



A remarkable property of a Gaussian beam is that it maintains its shape as it propagates through space. It simply expands, with angle θ, according to the following expression

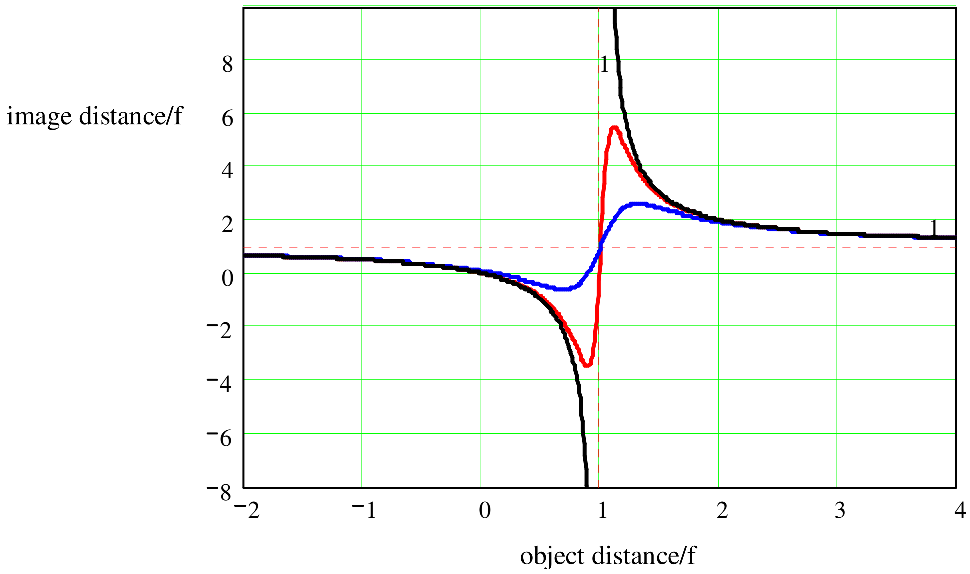
$$\theta = \frac{\lambda}{\pi \cdot w_o} \tag{2}$$

where λ is the wavelength.



The wavefront of the expanding beam has a convex curvature whereas a beam being focussed has a concave wavefront. At the beam waist the wavefont is planar – a bit like the collimated beam formed in geometrical optics by a point source situated at the front focal plane.

The plot below shows the normalised image distance against normalised object distance for a lens, of focal length f=10mm, performing a Gaussian beam transformation. Note that the distances have been normalised by the focal length of the lens. The black curve represents the geometrical optics case The red and blue curves correspond to Gaussian beams of input waists 15um and 25um respectively.



For the geometrical case it can be seen that the image appears at infinity when the object is at the front focal plane and vice versa, as expected from the standard imaging formula:

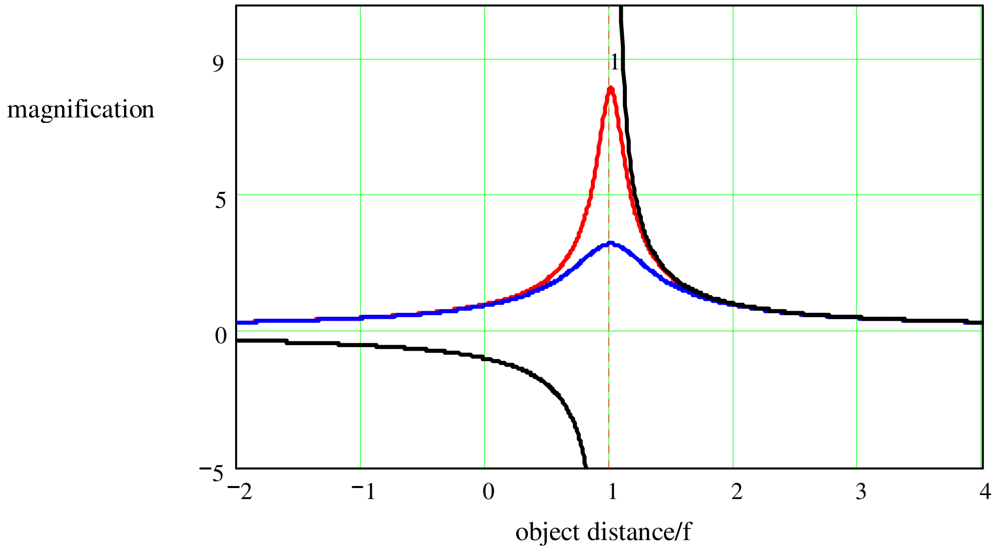
$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \tag{3}$$

where u and v are the object and image distances, respectively.

For Gaussian beams, however, when the object waist is at the front focal plane of the lens then the image waist actually occurs at the back focal plane – very strange!

In fact, as the object distance is increased the image distance also increases at first, but then reaches a peak and heads back towards the lens. It can be seen that the position of this peak depends on the particular beam waist in question.

Finally, the smaller the beam waist the more closely it resembles the geometrical case. Now let’s have a look at the corresponding magnification curves:



Here we see that when the object beam waist is at the focal length the magnification is greatest and falls off as the object is moved either closer to the lens or further from it. Again, the smaller the beam waist the more like the geometrical case it becomes.

Now, it will be seen from the first plot that for each image distance there are two possible object distances (except for the case when the object waist is at the front focus). The lower plot shows that different object distances will give different magnifications. Thus there are two possible image waists that occur at the same image distance, depending on where the object waist is located.

Regarding **coupling into a fibre**, it is well known that in geometrical optics a lens of any focal length lens can be used to give the required magnification if the object and image distances are chosen correctly. Not so with Gaussian beams – the focal length of the lens has to be greater than a characteristic length, fo, which is given by:

$$\mathrm{fo} = \frac{\pi \cdot \mathrm{w1} \cdot \mathrm{w2}}{\lambda} \tag{4}$$

where w1 and w2 are the object and image beam waist radii, respectively.

The efficiency of coupling into a single-mode fibre may calculated from the overlap of the complex field functions of the incident Gaussian beam and the beam waist of the field in the fibre (known as the ‘mode-field diameter‘). The ‘complex’ notation simply includes information about the curvature of the Gaussian wavefront as well as the width of the incident beam at the fibre end.

It was seen in eqn. (2) that the beam waist and the angle of divergence of a Gaussian beam are inter-related. This means that in order to achieve maximum efficiency it is only necessary to match one of these parameters to the corresponding one in the fibre. Thus optimum coupling is achieved when the incident beam waist is focussed at the end of the fibre and is the same size as the mode-field diameter of the fibre. This is in contrast to the geometric case of coupling into multimode fibre, where the angular extent of light, known as the fibre’s numerical aperture, is determined by the refractive index difference between the core and cladding region of the fibre and is not related to its diameter. Thus for efficient coupling in a multimode fibre both the angle and width of the incident beam have to be independently matched to the fibre’s parameters.

For more information on Gaussian beams and their peculiar properties see: “Focussing of Spherical Gaussian Beams” by Sidney A Self, Applied Optics, vol.22, no.5, March 1983.