## MPX Image Analyser - User Guide

## 1 Introduction

The MPX Image Analyser (MIA) reads in data that has been measured on an MPX Modal Explorer. It then computes the Mode Power Distribution (MPD) and uses this to compute the loss between two offset fibres.

Version 1.0 of MIA has the following restrictions:
Assumes circular fibres
Assumes both fibres are the same diameter and NA.

## 2 Software package

MIA is a Visual Basic (VB6) program and utilises reference mode-field data that has been previously computed in Mathcad. The required files for MIA to work are as follows:


The source Mathcad files are:
Laguerre.MCD
Overlap.MCD

## 3 Software operation

Load and run EFSA.vbp, select 'load data' and choose an MPX-1 data file to read. The data will be computed and the screen should look like the figure below.

The image, top left, is the MPX bitmap.
EF is the computed EF, according to latest (2009) versions of IEC61280-1-4 and FOTP-203.
MPD is the Mode Power Distribution, computed from the near-field intensity profile, according to FOTP220.

NF shows the measured near-field intensity profile in red, taken as the average of four diameters at $45^{\circ}$ angles through the bitmap image. The green profile is the near-field regenerated from the MPD. This is given as a diagnostic to check that the MPD is correct. Selecting the checkbox shows the difference between the red and green curves, magnified by x10.


Loss shows the computed loss due to offset between two identical fibres, where the source (input) fibre has the MPD shown above. Note that losses due to Fresnel reflection at the fibre interfaces are not included in the model. A cursor is provided to read the loss vs offset data.

The 'save data' button allows the computed loss data and to be saved to disc. This is identified by the file extension .LOSS.

The 'quit' button exits the software.

## 4 Detailed operation

### 4.1 Compute the Laguerre-Gauss modes in Mathcad

The Mathcad program 'Laguerre.MCD' computes the complete guided mode-set for a parabolic gradedindex fibre. The electric field $E$ as a function of mode number $(\eta, \mu)$ is given by the time-independent Helmholtz equation, as follows:

$$
\begin{equation*}
E_{\eta, \mu}(\rho, \phi, z)=\rho^{\eta} e^{\frac{-\rho^{2}}{2}} L_{\mu-1}^{\eta}\left(\rho^{2}\right) \sin \left(\eta \phi+\theta_{o}\right) e^{-j(\beta z)} \tag{1}
\end{equation*}
$$

where $\phi$ is azimuthal angle, $z$ is position along the fibre and $\rho$ is a radial variable given by

$$
\begin{equation*}
\rho=\frac{\mathrm{r}}{\mathrm{a}} \sqrt{\mathrm{~V}} \tag{2}
\end{equation*}
$$

where a is the fibre radius and V is the normalised frequency, given by

$$
\begin{equation*}
\mathrm{V}=\mathrm{ak}_{\mathrm{o}} \mathrm{NA} \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
L_{\mu^{\prime}}^{\eta}\left(\rho^{2}\right)=\sum_{s=0}^{\mu^{\prime}} \frac{\left(\mu^{\prime}+\eta\right)!(-1)^{s} \rho^{2 s}}{(\eta+s)!\left(\mu^{\prime}-s\right)!s!} \tag{4}
\end{equation*}
$$

is the generalised Laguerre polynomial, where, for convenience, the substitution $\mu^{\prime}=\mu-1$ has been made. $\theta_{0}=0$ or $\pi / 2$, represents pairs of degenerate modes when $\eta \neq 0$, these are cosine and sine functions.

In the Mathcad program, 'Field1' and 'Field2' represent the complete set of cosine and sine mode-field solutions. These are saved to disc as 'Parafield1.prn' and 'Parafield2.prn'.

The default parameters are:

| Fibre diameter: | 50 um |
| :--- | :--- |
| Wavelength: | 0.85 um |
| NA: | 0.21 |
| Cladding index: | 1.457 |
| Grid size: | $121 \times 121$ points |
| Grid range: | 72 um |
| Grid spacing: | 0.6 um |

### 4.2 Compute the mode overlap integrals in Mathcad

Mathcad program "Overlap.MCD' computes the overlap between each and every mode-field in the source (input) fibre with each and every mode-field in the target (output) fibre.

Firstly, the mode-field solutions computed above are read in as Parafield1 and Parafield2. These are then offset across the $121 \times 121$ grid by an amount given by the parameter 'offset', which must be in multiples of 0.6 um, to match the grid spacing. The resultant mode-field sets are called P1_os and P2_os.

The mode overlap factor $\varepsilon$ is given by

$$
\begin{equation*}
\varepsilon=\frac{\iint E_{S}(x, y) E_{T}(x, y) d x d y}{\sqrt{\iint E_{S}(x, y) E_{S}(x, y) d x d y} \sqrt{\iint E_{T}(x, y) E_{T}(x, y) d x d y}} \tag{5}
\end{equation*}
$$

where $\mathrm{E}_{\mathrm{S}}$ and $\mathrm{E}_{T}$ are the mode-fields of the source and target fibres respectively.
The mode overlap factor can take values from zero, in the case of no overlap, to unity for complete overlap.

There are four subroutines to compute eqn. (5). These are of the form 'Overlap1(a,b)' and correspond to the overlap between the sine and cosine fields in the source fibre with the sine and cosine fields in the target fibre. The overlap factors are then summed into their corresponding mode groups to give OS1_sum which is the MPD of the target fibre A fully-filled source fibre is assumed for the source fibre, given by $M P D(m)=m$, where $m$ is the mode group number.

The calculation has been repeated for a range of offsets from zero to 7.8 um and the resulting MPD's are saved in a file called 'weights.csv' which is required by the VB program.

It has been found that is not necessary to carry out the overlap calculation for each source MPD used. Instead, the same loss result is obtained by multiplying the fully-filled MPD values in 'weights.csv' by the MPD of the source fibre. This means that the overlap integrals vs offset only have to be computed once to set up 'weights.csv'.

Data required for the VB program to work is saved at the foot of the Mathcad program. These are a 'design' matrix called 'X.prn' and an intermediate matrix called 'Inv.prn', these are required to compute the MPD from the measured MPX data.

### 4.3 Computation of the MPD in MIA

The total power in a mode group $m$ is given by

$$
\begin{equation*}
\mathrm{P}_{\mathrm{g}}(\mathrm{~m})=\sum_{\mu=1}^{\mathrm{int}[(\mathrm{~m}+1) / 2]}\left[\left(\mathrm{E}_{\eta, \mu}\right)_{\theta_{0}=0}^{2}+\left(\mathrm{E}_{\eta, \mu}\right)_{\theta_{0}=\pi / 2}^{2}\right] \tag{6}
\end{equation*}
$$

where $\eta=m+1-2 \mu$.

Then the total power in the fibre $P_{\text {tot }}$ is given by

$$
\begin{equation*}
\mathrm{P}_{\mathrm{tot}}=\sum_{\mathrm{m}=1}^{\mathrm{M}}\left[\mathrm{P}_{\mathrm{g}}(\mathrm{~m}) \cdot \operatorname{MTF}(\mathrm{m})\right] \tag{7}
\end{equation*}
$$

where $M$ is the total number of mode groups and $M T F(m)$ is the Mode Transfer Function and is given by

$$
\begin{equation*}
\operatorname{MTF}(\mathrm{m})=\frac{\operatorname{MPD}(\mathrm{m})}{\mathrm{m}} \tag{8}
\end{equation*}
$$

The MTF may be computed by matrix manipulation (FOTP-220), as follows

$$
\begin{equation*}
\operatorname{MTF}(\mathrm{m})=\left[\mathrm{P}_{\mathrm{g}}(\mathrm{~m})\right]^{-1} \mathrm{P}_{\mathrm{tot}} \tag{9}
\end{equation*}
$$

In order to improve the stability of eqn. (9) a method called 'Singular Value Decomposition' (SVD) is used in MIA, as follows

$$
\begin{equation*}
\mathrm{MTF}=\left[\mathrm{P}_{\mathrm{g}}{ }^{\mathrm{T}} \mathrm{P}_{\mathrm{g}}\right]^{-1}\left[\mathrm{P}_{\mathrm{g}}{ }^{\mathrm{T}} \mathrm{P}_{\mathrm{tot}}\right] \tag{10}
\end{equation*}
$$

In 'Overlap.MCD' the term $P_{g}(m)$ is given by 'X.prn' and the inverse term is given by 'Inv.prn'. $P_{\text {tot }}$ is the measured near-field intensity profile.

### 4.4 Computation of loss in MIA

The loss for a range of offsets i is given by

$$
\begin{equation*}
\operatorname{loss}_{\mathrm{i}}=10 \log \left[\frac{\sum_{\mathrm{m}} \mathrm{MTF}(\mathrm{~m}) \cdot \mathrm{MTF}_{-} \mathrm{FF}_{\mathrm{i}}(\mathrm{~m}) \cdot \mathrm{m}}{\left.\sum_{\mathrm{m}}^{\mathrm{MTF}(\mathrm{~m}) \cdot \mathrm{MTF}_{-} \mathrm{FF}_{0}(\mathrm{~m}) \cdot \mathrm{m}}\right]}\right] \tag{11}
\end{equation*}
$$

where $\operatorname{MTF}(\mathrm{m})$ is the MTF computed in eqn. (10) and MTF_FF $_{\mathrm{i}}(\mathrm{m})$ is the MTF in the target fibre for an offset $i$ when the source fibre is fully-filled. A table of MTF_ $\mathrm{F} \overline{\mathrm{F}}_{\mathrm{i}}(\mathrm{m})$ is stored in the file 'weights.csv' for use by MIA. MTF_FF 0 ( m ) is the MTF in the target fibre at zero offset.

## 5 Default Parameters

The default parameters for the Mathcad computations are given in section 4.1. If any of these is changed it important to recompute the following:

The Laguerre mode-fields in 'Laguerre.MCD' and save as 'Parafield1.prn' and 'Parafield2.prn'
The overlap integrals in 'Overlap.MCD' and save as 'X.prn' and 'Inv.prn'. Also need to set up a new 'weights.prn' matrix.
and also change the constants in EFSA.bas, accordingly
Public Const MCnum = 120 'the size of the mode-field grid minus 1
Public Const MCrange $=36$ 'the half range of the grid in um
Public Const $\mathrm{Ng}=18 \quad$ 'the number of mode groups minus 1
Public Const fibrediam =50 'the fibre diameter in um

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